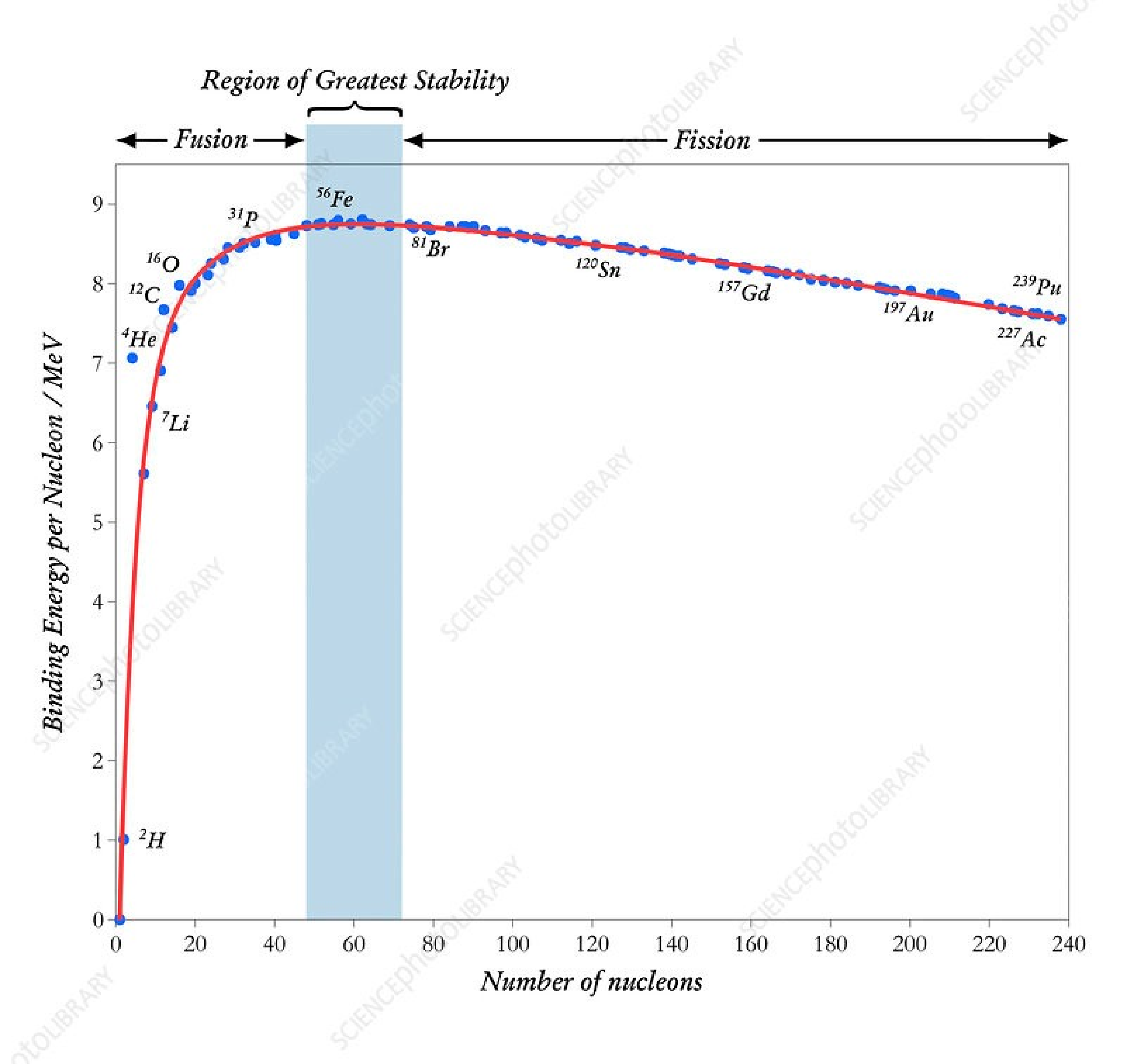
Új Nemzeti
Kiválóság Program

VII. Nuclear Fission and Fusion

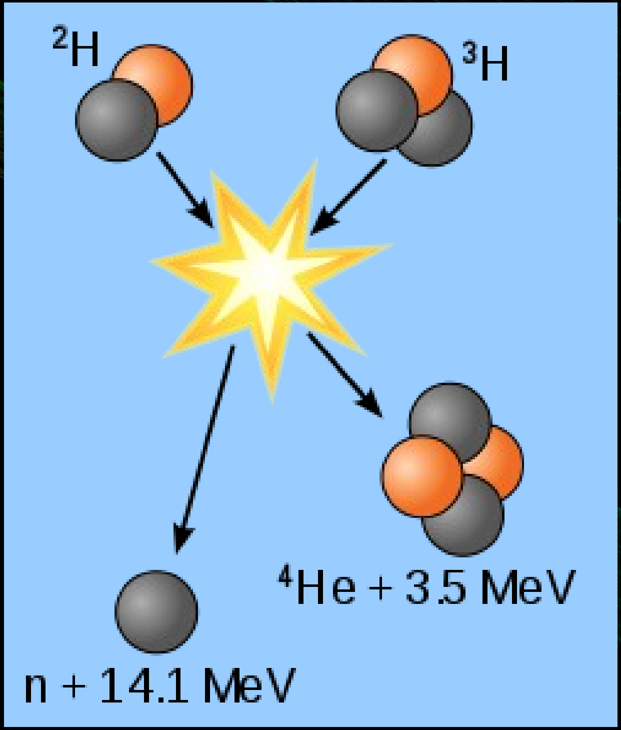
Lorant Csige
Laboratory for Nuclear Physics
Hungarian Academy of Sciences

Nuclear binding energy: possibility of energy production



Nuclear Fusion

- Some basic principles:
  - two light nuclei should be very close to defeat the Coulomb repulsion → nuclear attraction: kinetic energy!!
  - not spontenious, difficult to achieve in reality
  - advantage: large amount of hydrogen and helium + solutions without producing any readioactive product

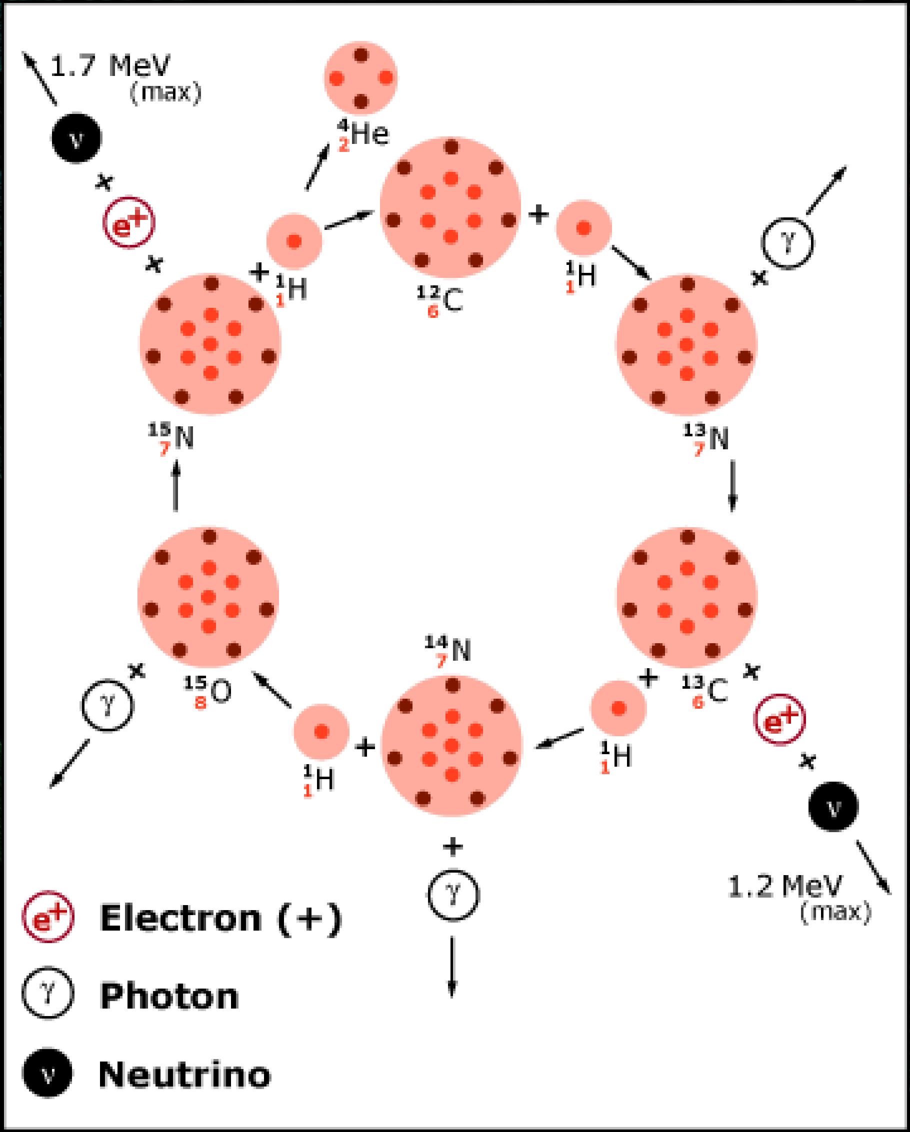


Nuclear fusion in stars

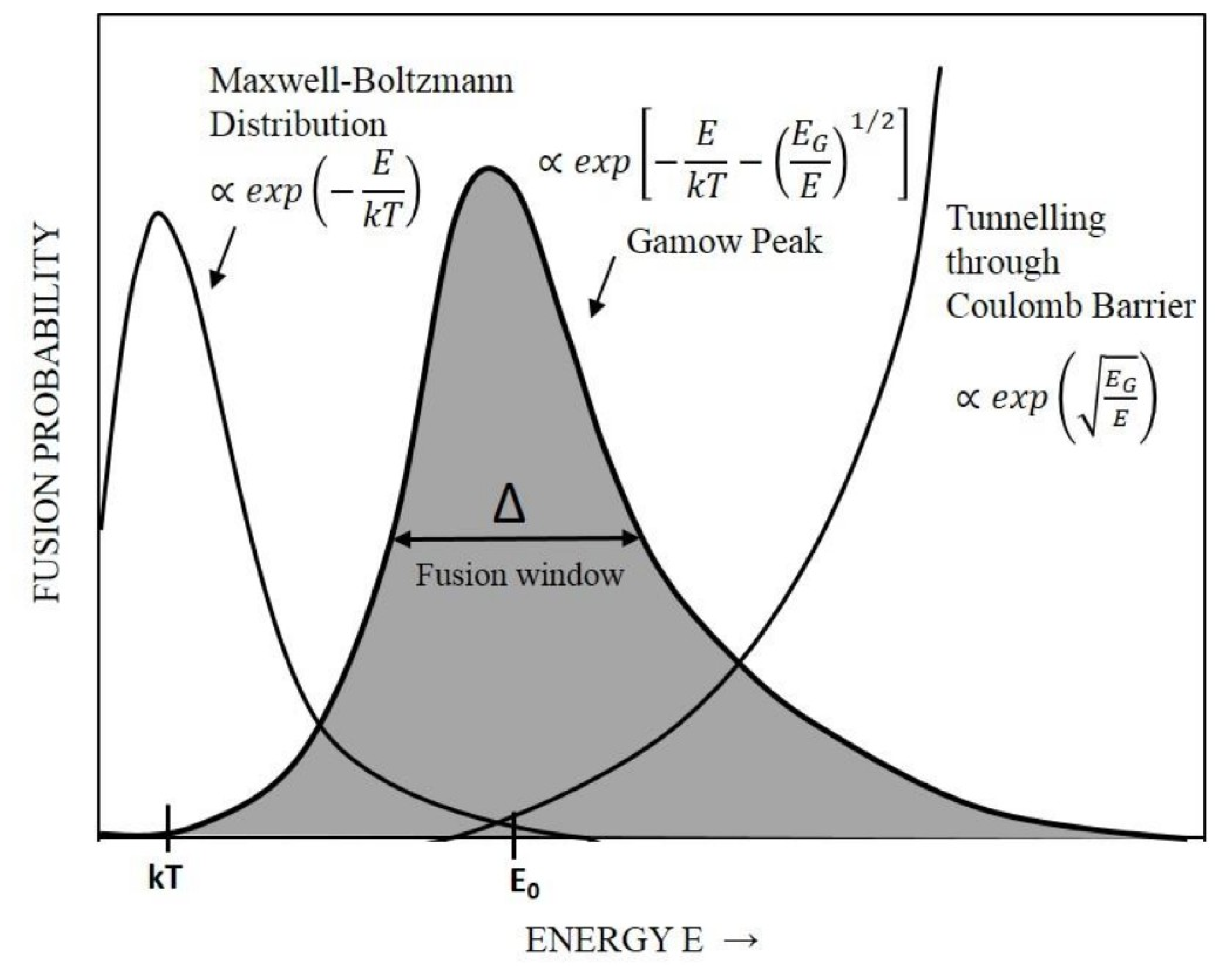
- proton-proton cycle: ~ mass of Sun, many branches
  - first step in all branches (Q=1.442 MeV): $p+p \rightarrow {}^2H+e^+ +\nu_e$
    - diproton formation (and immediate decay back to two protons) is the ruling process
    - stable diproton is not existing → proton – proton fusion with instant beta decay!
    - very slow process since weak interaction plays role → cross section has not yet been measrued experimentally (one proton „waits” 9 billion years to fuse)
  - second step: ${}^2H+{}^1H \rightarrow {}^3He + \gamma + 5.49$ MeV
    - very fast process: only 4 seconds on the avarage
    - ${}^3He$ than fuse to produce ${}^4He$ with three (four) differenct reactions (branches)
  - ppI branch: ${}^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H + 12.86$ MeV
  - ppII branch:
    - ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$
    - ${}^7Be + e- \rightarrow {}^7Li + \nu_e$
    - ${}^7Li + {}^1H \rightarrow {}^4He + {}^4He$
  - ppIII branch: only 0.11% energy of Sun, but source of neutrino problem
    - ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$
    - ${}^7Be + {}^1H \rightarrow {}^8B + \gamma \rightarrow {}^8Be + e^+ + \nu_e \rightarrow {}^4He + {}^4He$
  - ppIV branch (hypotetical): ${}^3He+{}^1H \rightarrow {}^4He+e^+ +\nu_e$

Nuclear Fusion in Stars

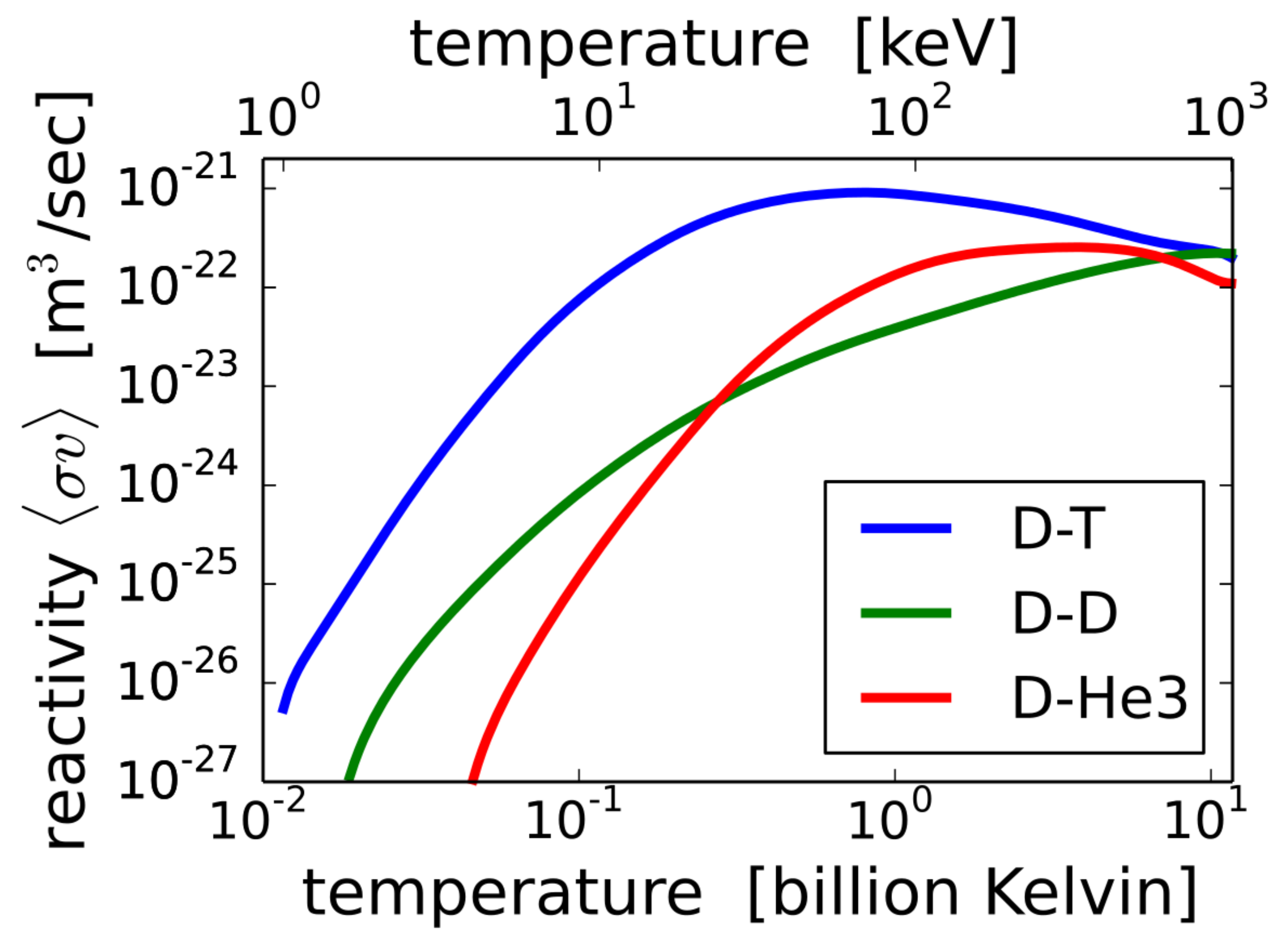
- CNO cycle: > 1.3 mass of Sun
  - protons fuse C, N, O are only catalysts
  - $4p + 2e^- \rightarrow {}^4He + 2e^+ + 2e^- + 2\nu_e + 3\gamma + 24.7\ MeV$
  - four different branches:
    - temperature (thus size) dependent



Fusion reaction in stars – Gamow window

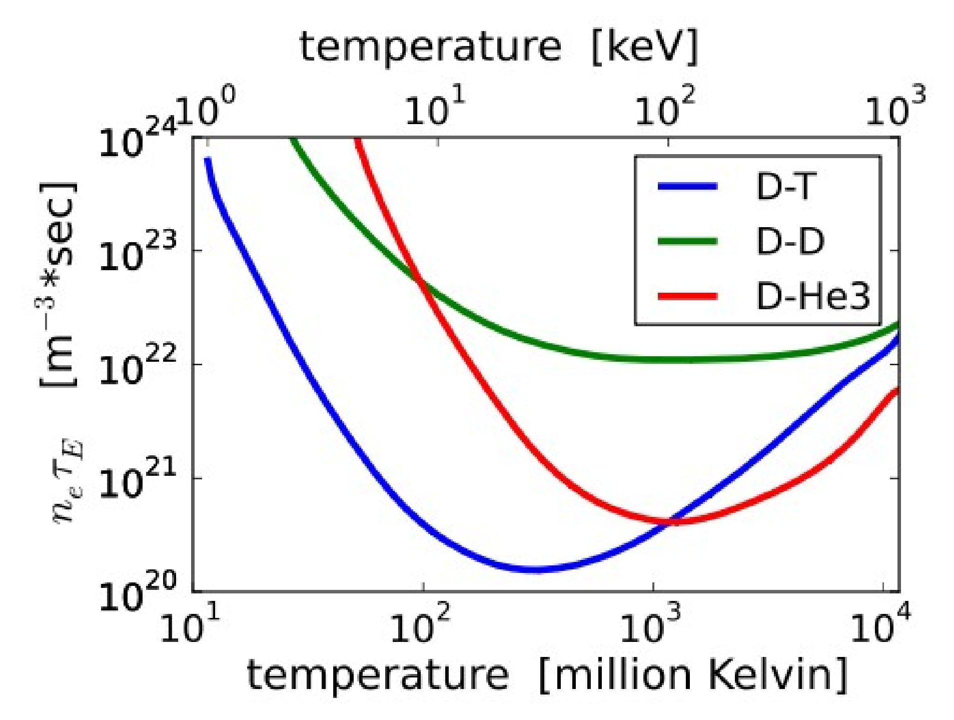


Fusion reaction in laboratory



Lawson criterion

- Energy balance of nuclear fusion:
  - rate of energy produced by fusion and energy losses in plasma
  - minimum condition to have a productive fusion
  - τ_E: confinement time of electron plasma



$$\frac{1}{4} n^2 \langle \sigma v \rangle E_{ch} \geq \frac{3 n k_B T}{\tau_E}$$

fusion reactions per volume per time

energy losses in plasma

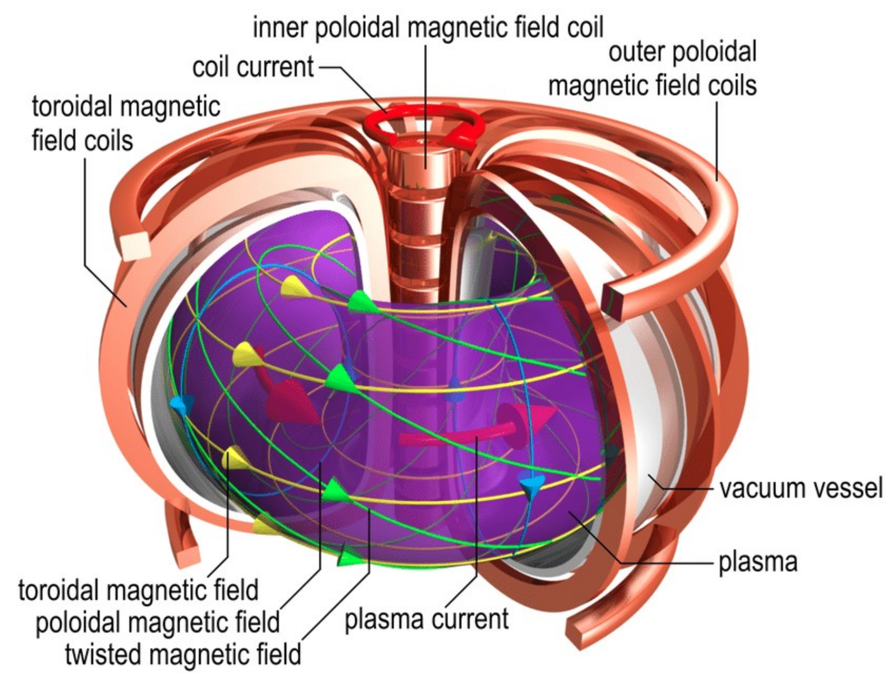
energy of the charged fusion products (3.5 MeV for D-T)

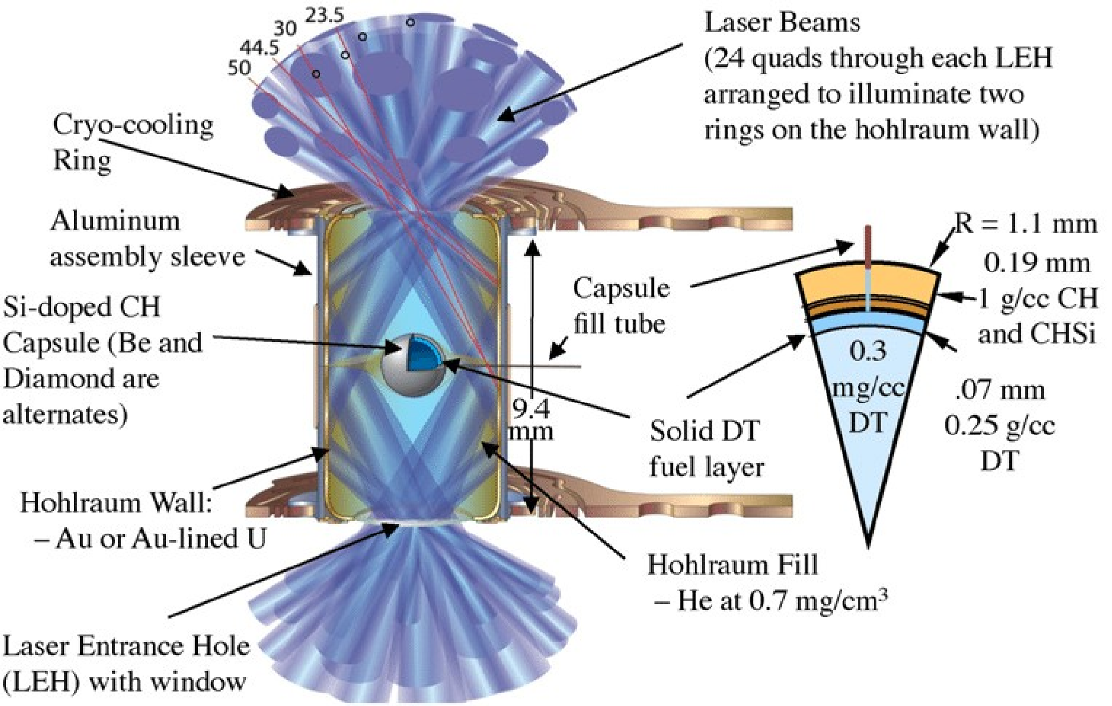
$$n \tau_E > 1.5 \cdot 10^{20} \frac{s}{m^3}$$

for D-T fusion at T=26 keV

Fusion reactors

- Two approched to fulfil the Lawson criterium:
  - inertial fusion by ultra-high power fast lasers (fast, high density plasma)
  - magnetic confinement (slow, low density plasma)





- TOKAMAK
- ITER
  - inertial fusion by ultra-high power fast lasers (fast, high density plasma)
  - magnetic confinement (slow, low density plasma)

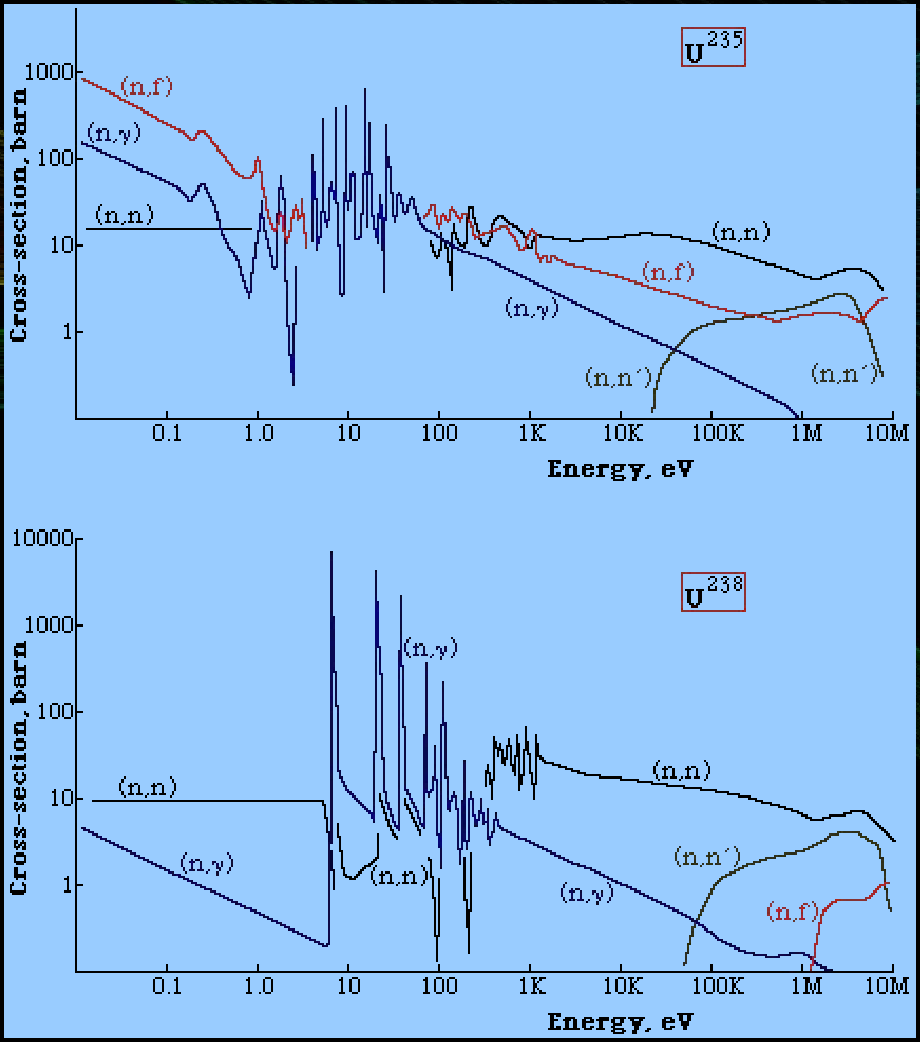
Nuclear fission - history

- First expectation: fission of heavy nuclei → energy is released
- Released energy is appear mostly as kinetic energy of the products
- From N/Z as a function of A → products are neutron-rich nuclei → β active
- Neutrons are emitted promptly and also some times later → delayed neutrons

Kinetic energy of fission fragments	168 MeV
Energy of fission neutrons	5 MeV
Energy of prompt gamma photons	7 MeV
Energy of particles from fragments	8 MeV
Energy of gamma photons from fragments	7 MeV
Energy of antineutrinos from fragments	10 MeV

- Properties, measurables in fission:
  - mass, charge, kinetic energy distributions of fission fragments
  - neutron multiplicity, neutron spectrum
  - number and energy of delayed neutrons
  - energy spectrum of prompt gamma photons

Fission cross sections of ^{235,238}U



- Fissile nuclei (^{235}U):
  - fission at thermal neutron energies
  - $\sigma_f \sim 1/v_n$ for thermal energies
- Fertile nuclei (^{238}U):
  - fission only at fast neutron energies (E_n > 1 MeV)
  - by thermal neutron capture, new fissile nuclei are produced:

    ^{238}U(n,γ) → ^{239}U(β-) → ^{239}Np(β-) → ^{239}Pu

    ^{232}Th(n,γ) → ^{233}Th(β-) → ^{233}Pa(β-) → ^{233}U

Theory of nuclear fission

- Bohr and Wheeler (1939): using liquid-drop model

$$Q_f = \Delta W_h + \Delta W_l - \Delta W_n$$

$$\Delta W = \alpha A \boxed{+\beta A^{2/3}} \boxed{-\gamma \frac{Z^2}{A^{1/3}}} - \xi \frac{(A/2-Z)^2}{A} + \delta A^{-3/4}$$

for asymmetric fission: $\frac{A_h}{A_l} = \frac{Z_h}{Z_l} = \frac{3}{2}$ ⇒ $A(Z)_h = \frac{3}{5} A(Z)$

if neglecting the last term: $Q_f = W_s + W_C - W_{sf} - W_{Cf}$

$$W_{sf} = \beta(3A/5)^{2/3} + \beta(2A/5)^{2/3} = 1.25\, W_s$$

surface energy is increasing in fission

$$W_{Cf} = 0.64\, W_C$$

Coulomb energy is decreasing

$$Q_f = 0.36\, W_C - 0.25\, W_s$$

- For ^{238}U fission it predicts $Q_f = 180$ MeV ($+Q_b = 20$ MeV)

Theory of nuclear fission: fission barrier

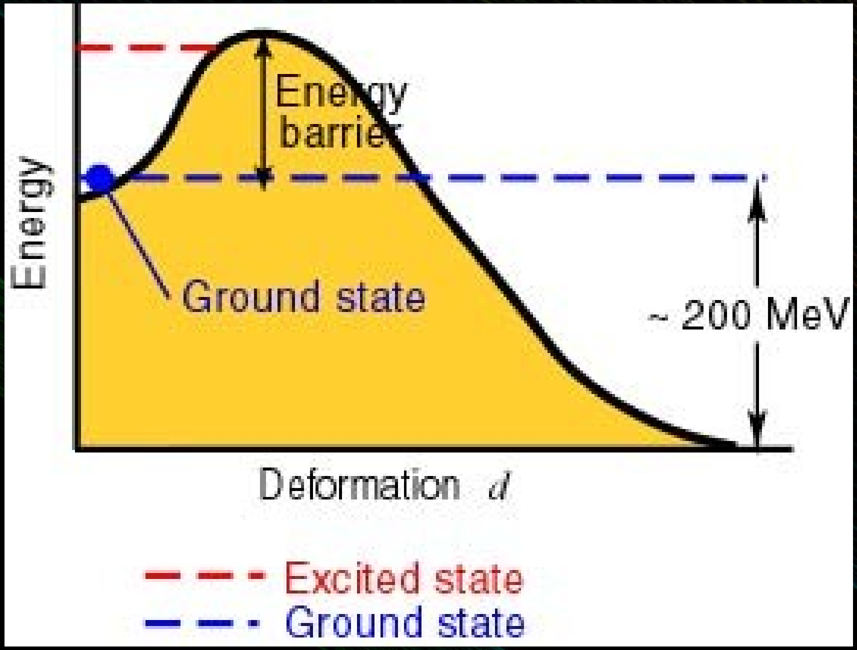
- So if $Q_f > 0$ fission is energetically favorable:

$$\frac{W_C}{W_s} = \frac{\gamma Z^2 / A^{1/3}}{\beta A^{2/3}} = \frac{\gamma}{\beta}\frac{Z^2}{A} > \frac{0.25}{0.36} = 0.7 \Rightarrow \frac{Z^2}{A} > 17$$

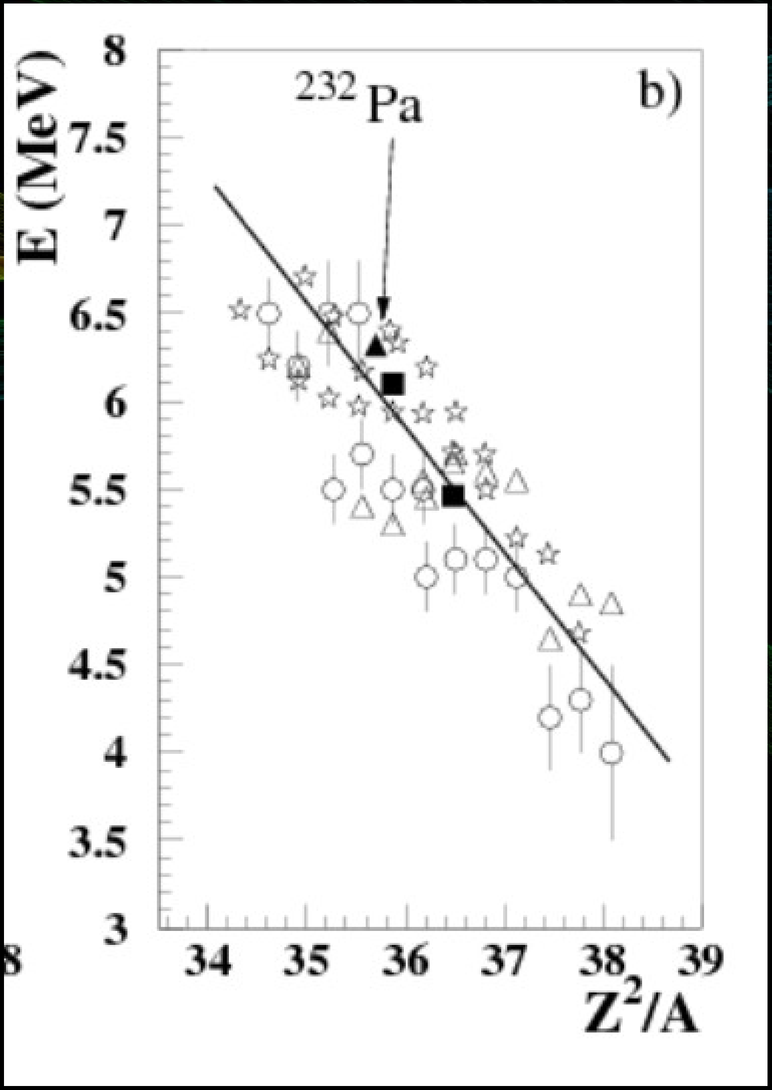
satisfied for $>{}^{108}\mathrm{Ag}$

Bohr-Wheeler fissility parameter

- Q_f is increasing with increasing fissility parameter
- However, fission was observed only for Th, Pa, U isotopes!!
- Energetically favorable DOES NOT mean energetically allowed
  - as in the case of alpha decay: energetically favored for heavy nuclei but classically forbidden by the Coulomb barrier (→ quantum tunneling)
- The mechanism: vibration is stabilized by the surface tension until vibration is large

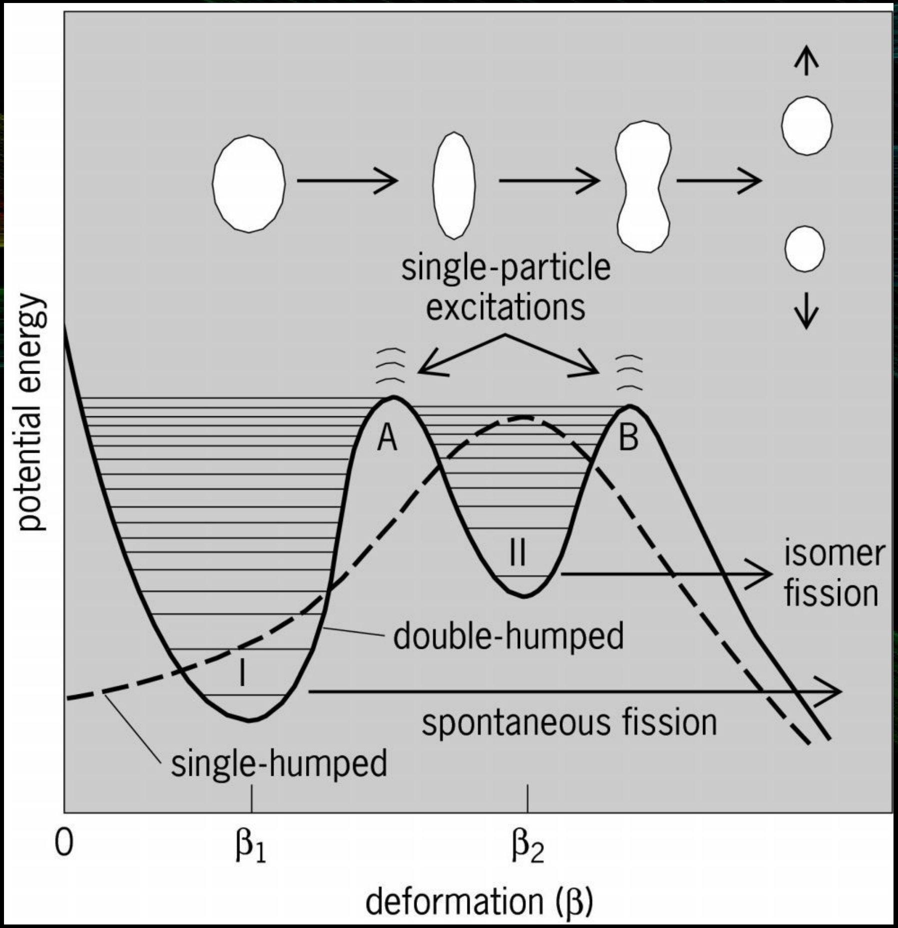


Fission barrier systematics



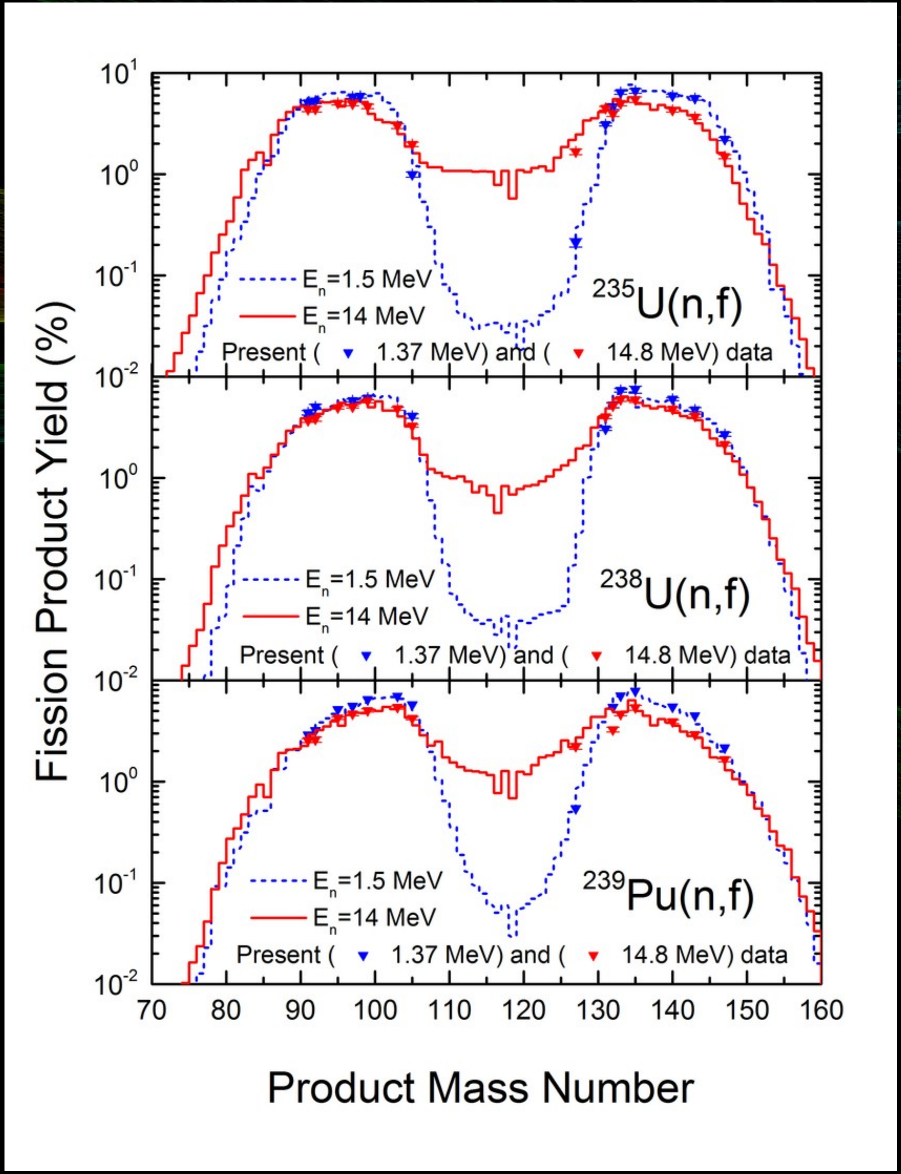
- For transuranium isotopes $Z^2/A > 49$ → fission barrier disappears!
- With increasing Z^2/A, the fission barrier is quickly increasing
  - spontanious fission through the fission barrier
  - long lifetime:
    - ^{238}U → $4.468 \cdot 10^9$ years
    - ^{252}Cf → 2.645 years
- At the region of the actinides the barrier height ~ 6-7 MeV → neutron separation energy

Theory of fission: fission barrier



- Fission barrier calculated by liquid drop model + microscopic shell corrections
- A 2nd local minimum appear for light actinides (Th, U isotopes)
  - isomeric states in the second potential minimum: highly deformed states with large excitation energy and long lifetime
  - isomeric (delayed) fission
    - investigation of strongly deformed nuclear states
  - gamma decay back is also possible

Mass distribution in fission



- At low excitation energy, the fission fragment mass distribution is very asymmetric
- Increasing excitation energy → the fragment mass distribution is getting symmetric

↓

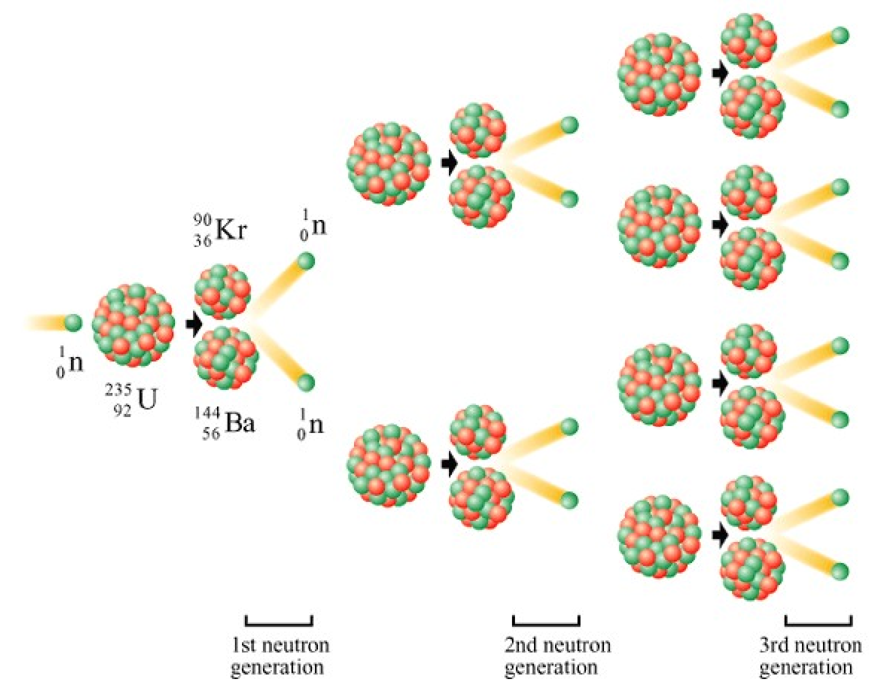
- One of the most compelling open question in the physics of fission:

Why asymmetric fission is favored?

- Possible explanation: **shell effects** controls the formation of fragments → no experimental proof yet… (only qualitive considerations)

Reactor physics - Basics

- In nuclear fission, energy is released as kinetic energy of the fragments (200 MeV) and neutrons are emitted



- Condition for self-sustainable energy production:
  - On the avarage, more than one neutron has to be produced in one fission
    - Avarage number of fission neutrons in ^{235}U: $<\nu_n>=2.5$
  - From the produced, more than one neutron has to induce fission

nuclear chain reaction

- Some terms in reactor physics:
  - multiplication factor k_{eff} and reactivity ρ:
    - if $k_{eff} < 1$: reactor is sub-critical ($\rho<0$) → reactor shutdown
    - if $k_{eff} = 1$: reactor is critical ($\rho=0$) → normal operation of a reactor
    - if $k_{eff} > 1$: reactor is supercritical ($\rho>0$) → starting a reactor, bomb
  - ρ measures how far we are from criticality

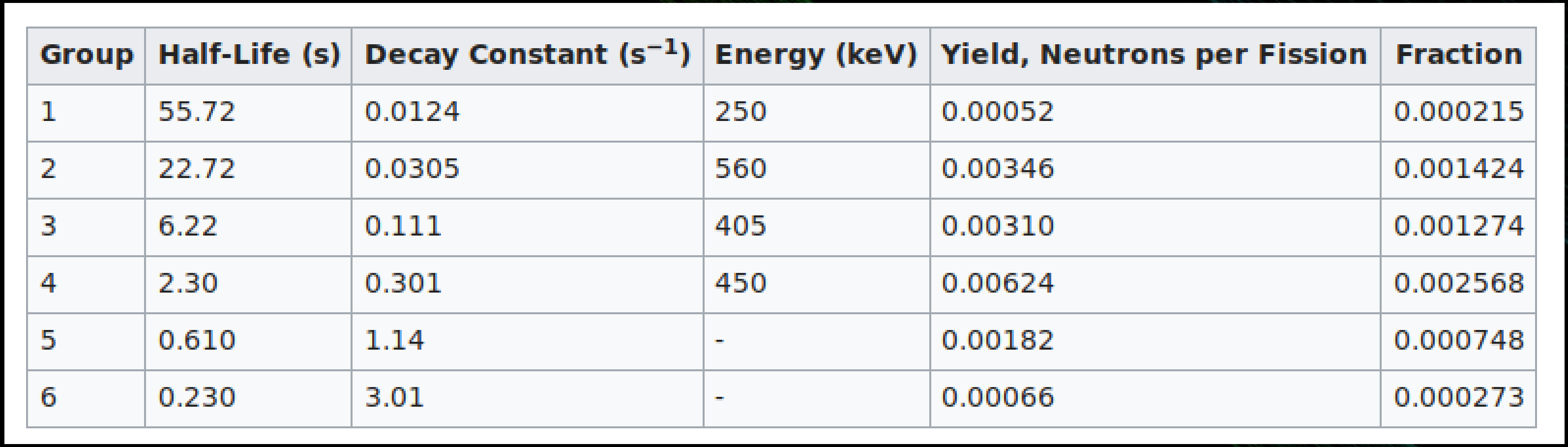
$$k_\infty = \frac{n_i}{n_{i-1}}$$

$$k_{eff} = k_\infty \cdot P_f \cdot P_t$$

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

Reactor physics - Control

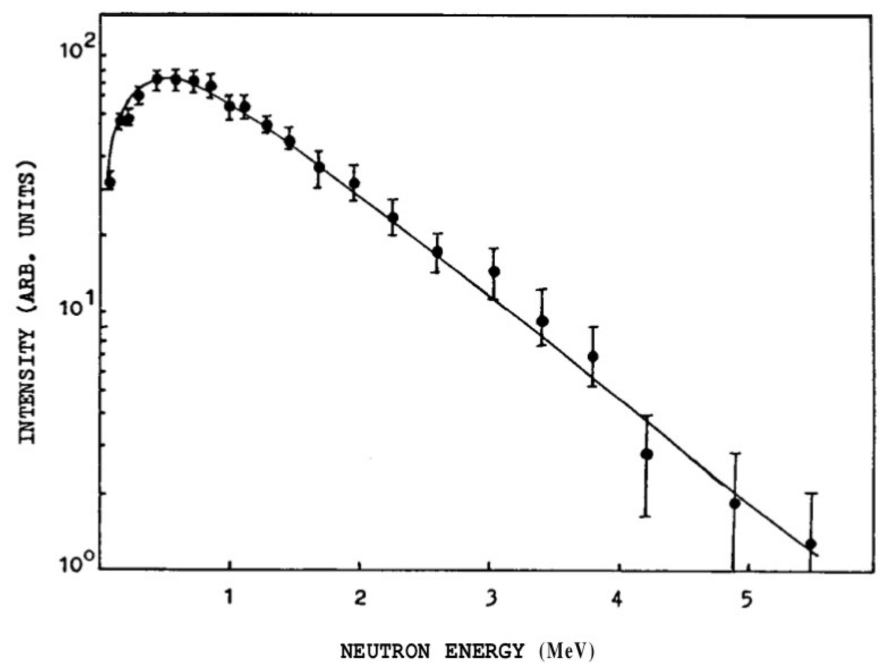
- Neutrons that are released immediately after the fission occurs are referred to as prompt neutrons → time scale of prompt neutrons „life” does not allow any interaction by the controls
- Fission products are radioactive → following radioactive decay, some daughter nuclei may have sufficient energy to release additional neutrons called delayed neutrons:
  - time scale of delayed neutrons „life” are determined by the half life of the radioactive fission product (2-3 seconds) → control is possible
- A reactor is designed to be sub-critical for prompt neutrons but critical for prompt neutrons + delayed neutrons
- Control is possible by absorbing delayed neutrons: ^{113}Cd and ^{10}B (fast *n*)



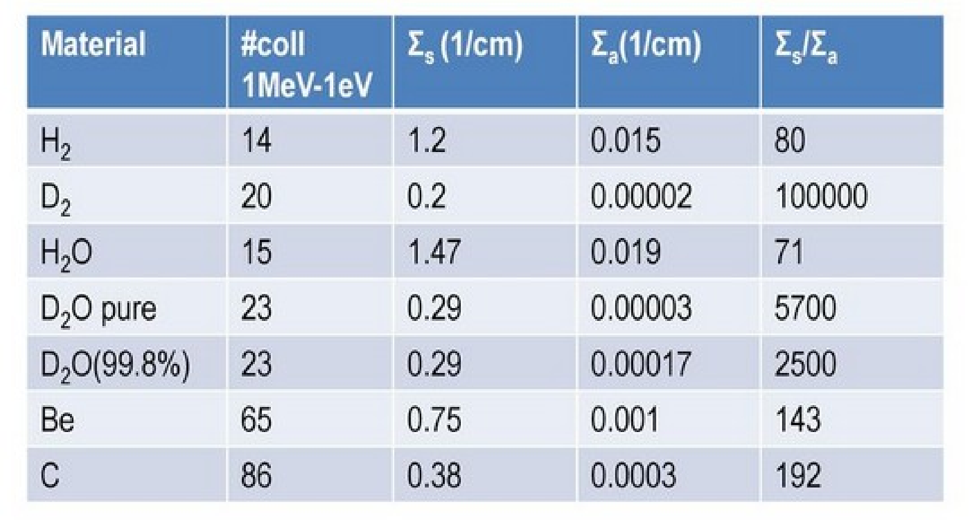
Group	Half-Life (s)	Decay Constant (s^{-1})	Energy (keV)	Yield, Neutrons per Fission	Fraction
1	55.72	0.0124	250	0.00052	0.000215
2	22.72	0.0305	560	0.00346	0.001424
3	6.22	0.111	405	0.00310	0.001274
4	2.30	0.301	450	0.00624	0.002568
5	0.610	1.14	-	0.00182	0.000748
6	0.230	3.01	-	0.00066	0.000273

Reactor physics: moderator

- Prompt neutron energy spectrum:
  - $<E_n>=0.7$ MeV
  - most of the prompt neutrons are fast neutrons → (n,f) cross sections is small!
  - we need to reduce the energy of the neutrons to thermal energies (< 1 eV)



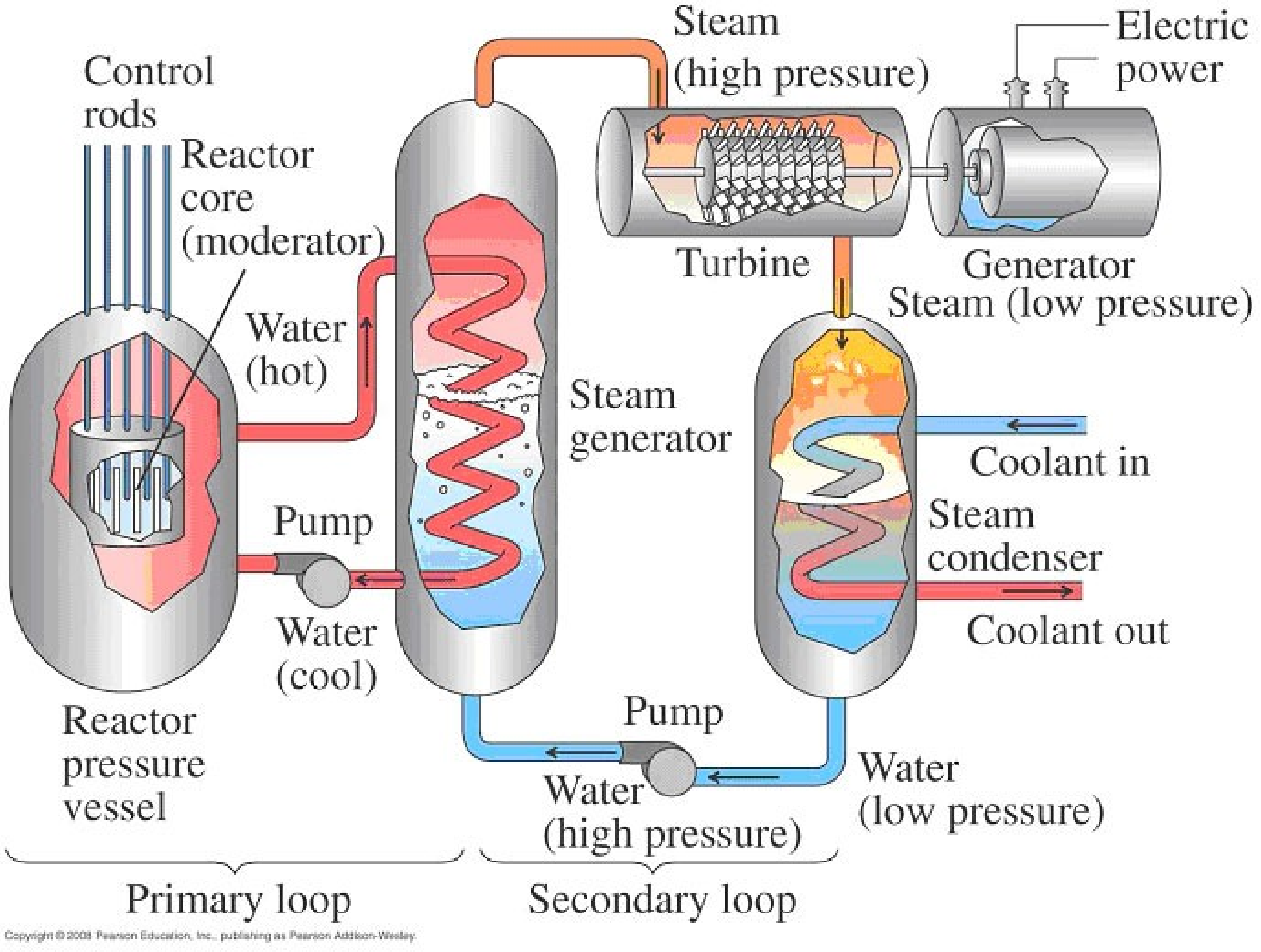
MODERATION



Material	#coll 1MeV-1eV	Σ_s (1/cm)	Σ_a(1/cm)	Σ_s/Σ_a
H_2	14	1.2	0.015	80
D_2	20	0.2	0.00002	100000
H_2O	15	1.47	0.019	71
D_2O pure	23	0.29	0.00003	5700
D_2O(99.8%)	23	0.29	0.00017	2500
Be	65	0.75	0.001	143
C	86	0.38	0.0003	192

- A good moderator is:
  - slowing down neutrons quickly (i.e. in a few collisions)
  - week neutron absorbtion

Reactor physics



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